Newtonian Jet Stability: The Role of Air Resistance

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The stability of high speed laminar Newtonian jets is studied as a function of ambient air pressure. For Weber numbers less than 5.3 (based on air density) air pressure has no effect on stability. Ambient viscosity, through the effect of shear stresses acting on the jet surface, gives rise to the maximum in the breakup curve. For large Weber numbers ambient pressure effects can alter, and eventually control, the appearance of the maximum.

Liquid jets ejected from long capillaries break up into droplets in a manner determined by the physical properties of the liquid and the ambient medium. In an earlier study (*2*) jets of Newtonian fluids were ejected into stagnant air at atmospheric pressures, and attention was confined to the properties and dynamics of the jet. In the present work we examine the role played by the ambient medium in destabilizing the jet, and in particular, the effect of ambient pressure.

Jet stability is usually characterized through a breakup curve, which shows the dependence upon jet velocity of the coherent length of the jet. At low velocities no dependence on ambient pressure is observed, and the breakup length increases linearly with velocity. As the jet velocity is increased the effect of the ambient medium is observed in the appearance of a pressure dependence of the breakup curve. A maximum breakup length occurs for each pressure, the low pressure curves being the more stable. All these observations seem quite reasonable, and one might think that the effect of the ambient medium is well established in view of the long history of studies of jet stability. (For a historical survey, see reference *1*). It is a fact, however, that no breakup data exist from which one may determine quantitatively the effect of the ambient medium on jet stability.

Two theories exist which have relevance to this study. Weber (*11*) examined the stability of a laminar Newtonian jet ejected into a stagnant inviscid atmosphere. The effect of the interaction between the jet and its surroundings is the establishment of a pressure distribution which accelerates the growth of certain disturbances. Weber's theory predicts the existence of a maximum in the breakup curve and attributes the existence of the maximum to aerodynamic pressure effects. Grant's experiments (*2*) showed that Weber's theory is not quantitatively accurate, and, in fact, is quite bad under typical experimental conditions.

Tomotika (*10*) examined the stability of a stationary cylinder of liquid surrounded by a second liquid, both liquids being viscous. Thus the viscosity of the ambient medium was accounted for. However, since the jet was assumed stationary, the effect of viscous drag on the jet was not accounted for, and Tomotika's theory would lead to a linear breakup curve showing no maximum. Tomotika's theory does allow an estimate of the importance of the viscosity of the ambient medium to be made, and Meister and Scheele (*6*) have recently reviewed this work.

In considering the interaction of a jet with its surroundings, and in trying to plan a well-defined experimental program, three factors seem to demand particular attention. According to Weber's theory, ambient pressure gives rise to the maximum in the breakup curve. One can test the significance of ambient pressure effects by injecting into surroundings maintained at various pressure levels.

According to Tomotika's work the jet-ambient kinematic viscosity ratio is significant. In the case of liquid jets in a gas maintained at pressures of the order of atmospheric or smaller, the ratio of kinematic viscosities is large compared to unity. The effect of the viscosity ratio is predicted to be nearly insignificant in this region. It would appear that one needs to work with liquid-into-liquid jets in order to study ambient viscosity effects with any sensitivity.

The vapor pressure of the liquid will give rise to a certain amount of evaporation into the ambient medium. This will be a complex process, with mass transfer strongly affected by the fluid dynamics near the liquid-gas interface. If the ambient pressure is below the vapor pressure of the liquid, evolution of bubbles, or flashing, may lead to violent disruption of the jet (*5*).

In this paper only the effect of ambient pressure variation will be investigated. Liquids will be used having vapor pressures sufficiently low that evaporation will be insignificant.

EXPERIMENTAL PROCEDURE

The experimental system is described in an earlier paper (*2*). Briefly, jets were produced by ejecting fluid from a pressurized reservoir through horizontal hypodermic tubes. A rapid flash photograph allowed measurements of breakup lengths. The tubes were sufficiently long that fully developed flow was achieved in all cases. Glass tubes enclosed the jet, and could be evacuated to ambient pressures as low as 0.007 atm. Procedures were as described earlier (*2*) and detail is given elsewhere (*3*).

The fluids used in this study were Dow Corning silicon oils, which are polydimethylsiloxanes. They were chosen because of their very low vapor pressure (less than 10^{-4} atm. at room temperature), and because of their availability in a wide range of viscosities.

Because these fluids are polymeric, the possibility of non-Newtonian and viscoelastic phenomena must be investigated. Figure 1 shows the viscosity of the most viscous fluid used in this study, plotted as a function of shear rate. Only at the highest shear rates is there a slight departure from newtonian flow.

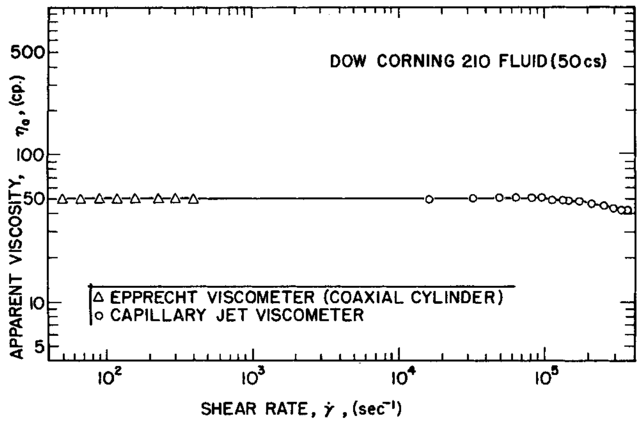

Fig. 1. Viscosity-shear rate behavior of the most viscous fluid studied.

The absence of significant viscoelastic effects was demonstrated by comparing the expansion-contraction behavior of the silicon oils with established results for Newtonian, inelastic fluids (*8*). It is known that Newtonian fluids upon ejection from a long capillary undergo a diameter change which is a function of the Reynolds number. Viscoelastic fluids, on the other hand, reach a diameter which is greater than that of a Newtonian fluid of comparable viscosity, and this increased diameter has been used as a measure of normal stresses developed within the capillary (*7,9*). Figure 2 compares the behavior of the most viscous silicone oil used with the Newtonian correlation, and indicates an absence of significant normal stresses. [For a study of stability of viscoelastic jets, see Kroesser and Middleman (*4*)].

Since the jet diameter differs from the capillary diameter, it is clear that the jet velocity also differs from the mass average velocity within the capillary. These changes occur very rapidly, over distances that are small compared to a breakup

TABLE 1. SOLUTION AND CAPILLARY PROPERTIES

Solution	Viscosity (Poise)	Density g./cc.	Surface tension dynes/cm.
1	0.49	0.96	20.8
2	0.34	0.95	20.5
3	0.20	0.95	20.3
4	0.047	0.92	19.3
5	0.013	0.85	18.2

All measured at 25°C.

Capillary	Diameter cm.	Length cm.
1	0.0833	4.33
2	0.0864	7.78
3	0.0833	40.0

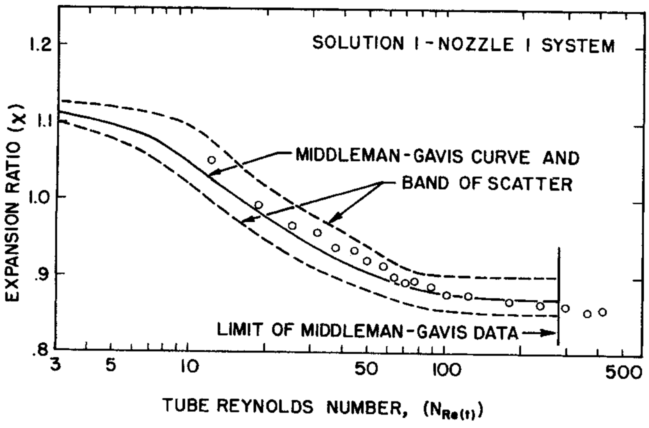

Fig. 2. Test for elastic effects in the most viscous fluid studied.

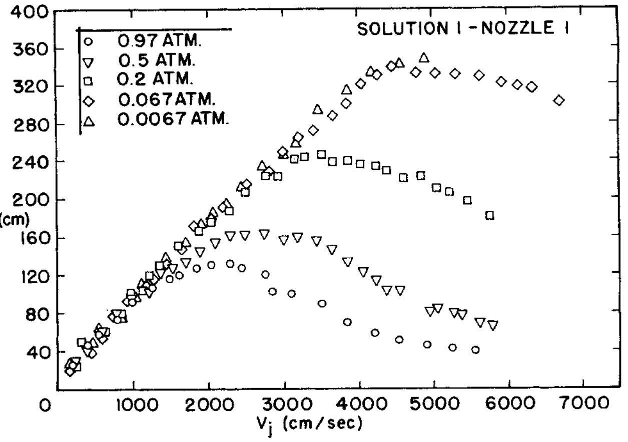

Fig. 3. Breakup curves as a function of ambient pressure.

length. Hence, most of the wave growth occurs on a jet whose diameter differs from the capillary diameter. In all of the parameters that characterize the breakup behavior, the jet diameter and jet velocity have been used. The jet diameter was obtained through the use of Figure 2, and the velocity follows from application of the continuity equation.

Table 1 gives the pertinent properties of the fluids studied, along with the dimensions of the capillaries used.

RESULTS

The breakup curves are presented as Figures 3 through 7. Each set of data represents the effect of ambient pressure at a fixed level of viscosity. It would appear that the effect of ambient pressure depends upon the viscosity of the fluid. This is only an apparent effect, however. One would expect the action of ambient pressure to be determined by the dynamic pressure $\rho_A V_j^2$, relative to the internal pressure $2\sigma/d_j$. Hence the critical velocity V_c, defined as that velocity at which the breakup curve deviates from the main body of data by 5% along the velocity axis, should satisfy the relationship

$$\frac{\rho_A V_c^2 d_j}{\sigma} = N_{We(A)} = \text{constant} \tag{1}$$

and a critical Weber number, based on ambient density, must be exceeded before breakup curves, at a given viscosity, show an ambient pressure dependence.

Since the critical velocity is a key parameter in describing the breakup curve it is most instructive to test Equation (1) by solving for V_c, to give

$$V_c = K\,(\sigma/d_j)^{1/2}\,(\rho_A)^{-1/2} \tag{2}$$

In all of the data examined, the surface pressure σ/d_j is essentially constant. Hence, if the concept of a critical

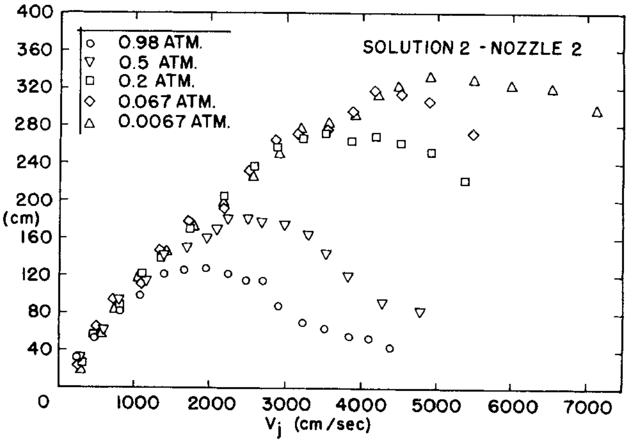

Fig. 4. Breakup curves as a function of ambient pressure.

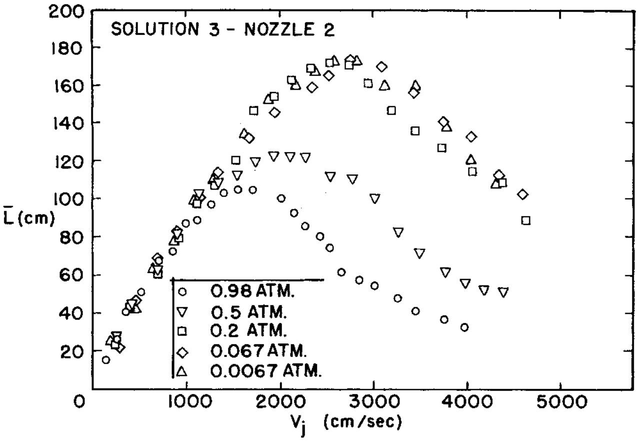

Fig. 5. Breakup curves as a function of ambient pressure.

Weber number is correct, V_c should be proportional to $\rho_A^{-1/2}$. This hypothesis was tested in Figure 8 with the ten data points for which a critical velocity V_c could be defined. A least squares fit of the data gave a line with a slope (with 95% confidence limits) of -0.652 ± 0.16. This confidence range includes the theoretical value of -0.5.

The relatively large confidence range reflects the influence of three scattered data points. If one performs a least squares analysis excluding these three points, the resultant slope is found to be -0.542 ± 0.069. The correlation coefficient (0.994) for these data is larger than that for the first correlation (0.957), indicating that a stronger linear correlation exists in the second case. With this as a statistical basis, a least squares line of slope -0.5 was put through the data, from which a critical Weber number was calculated as

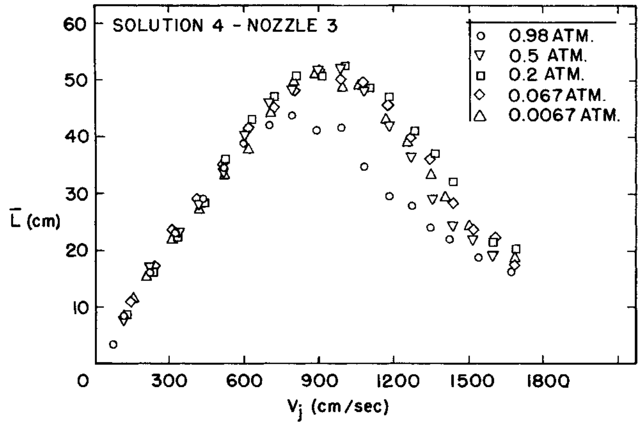

Fig. 6. Breakup curves as a function of ambient pressure.

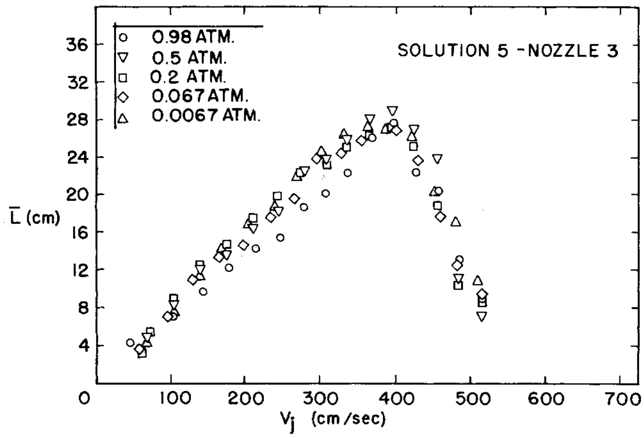

Fig. 7. Breakup curves as a function of ambient pressure.

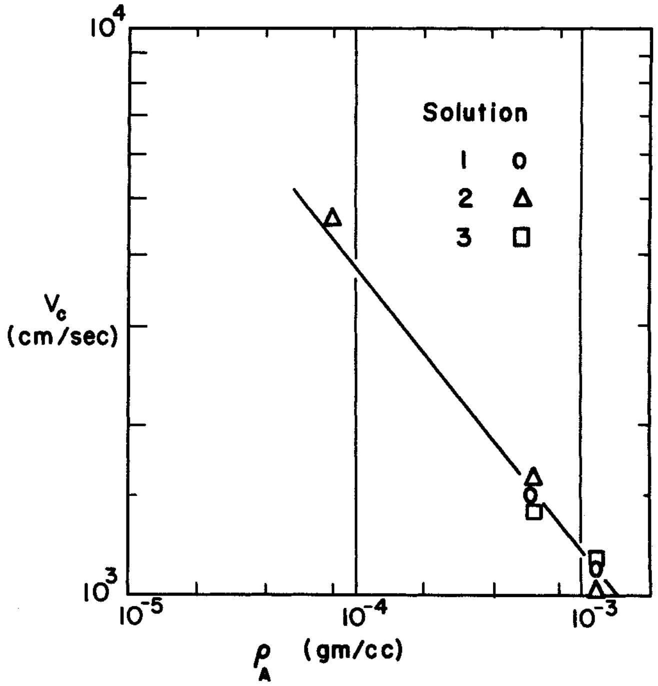

Fig. 8. Critical velocity data plotted according to Equation (2).

$$N_{We(crit.)} = 5.3 \tag{3}$$

Jets having Weber numbers below 5.3 should show no effect of ambient pressure.

Since some of the breakup curves exhibit maxima independent of ambient pressure, it does not seem possible to argue, with Weber (*11*), that the maximum is caused by ambient pressure effects. This leads to the conjecture that the maximum is caused by ambient viscous effects, but perturbed by pressure effects.

A characterization of the maximum may be obtained by examining the behavior of the maximum in the breakup curve, for those curves free of ambient pressure effects. Figure 9 shows the jet Reynolds number, based on the velocity at the maximum, as a function of the ratio of jet to ambient viscosities. It should be noted, of course, that the ambient viscosity was not varied, and so the generality of the dimensionless plot is not established for an ambient medium other than air at atmospheric pressure or lower. It is concluded, from these results, that ambient viscous forces give rise to the maximum in the breakup curve. This is contrary to the notion of Weber that the maximum is caused by ambient aerodynamic pressure effects.

If the Weber number is sufficiently large, the maximum in the breakup curve depends on ambient pressure. Fig-

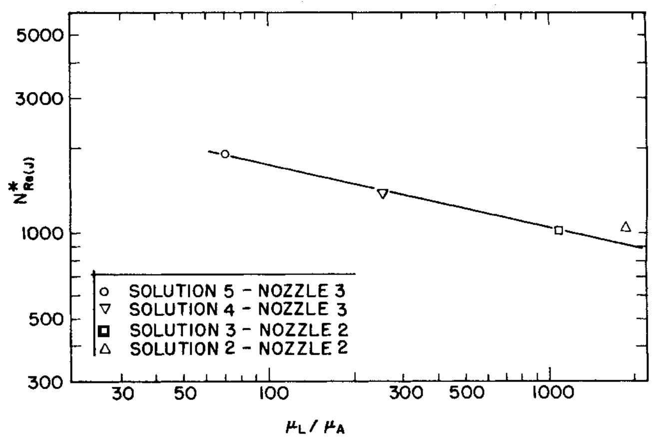

Fig. 9. Correlation for the maximum velocity in data free of ambient pressure effects.

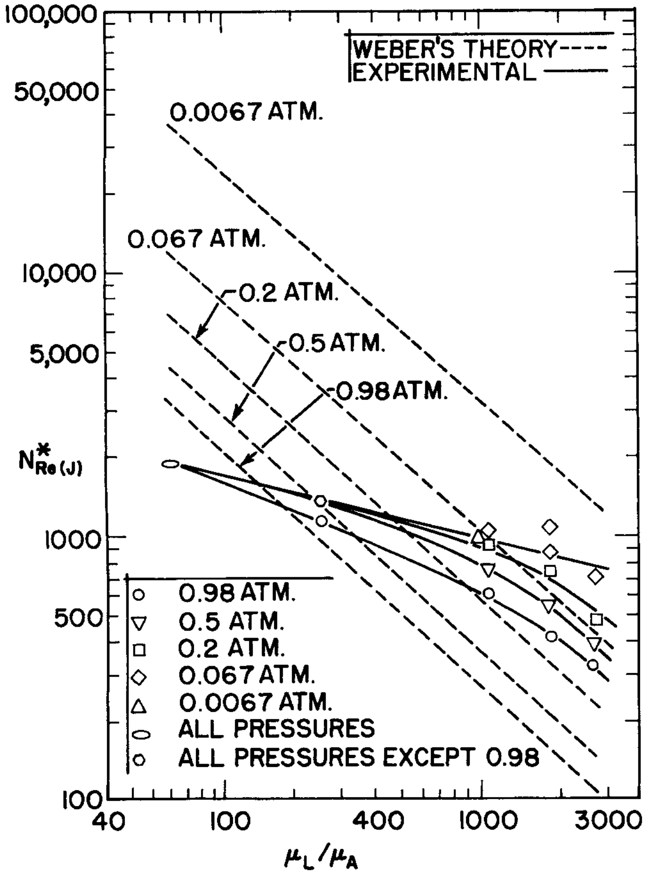

Fig. 10. Correlation for the maximum velocity in all data.

ure 10 shows the Reynolds number, based on the velocity at the maximum in the breakup curve, as a function of μ_L/μ_A, with ambient pressure as a parameter. Ambient pressure causes the maximum to occur at a smaller Reynolds number than in jets free of aerodynamic pressure effects. Figure 11 shows the behavior of the maximum breakup length L^*.

Also shown in Figures 10 and 11 are the predictions of Weber's aerodynamic theory. This theory does not consider μ_A, but does predict a dependence on μ_L. Hence, the apparent dependence on μ_A is just the dependence on μ_L, at fixed μ_A. While Weber's theory might be valid in the limit of large μ_L/μ_A, the data do not permit any conjecture to be made on this point.

If one examines photographs to observe the types of waves which destroy the jet, it may be seen that jets with Weber numbers (based on ambient density) less than 5.3 are destroyed by axisymmetric waves leading to drop formation. This is so even beyond the maximum in the breakup curve, contrary to the notion that the maximum is associated with a change in breakup mechanism from axisymmetric to transverse waves.

On the other hand, jets affected by ambient pressure do show a shift in breakup mechanism in the neighborhood of the maximum. Hence, jets of sufficiently large Weber numbers are significantly affected by ambient pressure driven transverse waves, and ambient viscosity effects become relatively unimportant.

CONCLUSIONS

The maximum commonly observed in breakup curves characteristic of liquid jets ejected into stagnant air is generated by viscous stresses as well as by aerodynamic pressure forces.

For Weber numbers (based on ambient density) less than 5.3 aerodynamic pressure forces have no effect, and breakup curves are independent of ambient pressure. The jet is broken by symmetric disturbances, even in the region of velocity beyond the maximum in the breakup curve. The appearance of the maximum is due to the effect of the shear stresses generated by the motion of the jet interface.

For larger Weber numbers aerodynamic pressure forces become important and lead to reduced stability of the jet. The maximum breakup length that can be achieved is reduced by pressure effects and becomes less dependent on the viscosity ratio of the two phases. In the neighborhood of the maximum the mechanism of breakup changes from symmetric to transverse wave growth.

ACKNOWLEDGMENT

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NOTATION

d_j = jet diameter, cm.
d_t = diameter of capillary, cm.
$\overline{L}$ = breakup length, cm.
N^*_{Re} = $V_j^*\ d_j\ \rho_L/\mu_L$ = Reynolds number based on velocity at maximum in breakup curve
$N_{We(A)}$ = $d_j\ V_j^2\ \rho_A/\sigma$ = Weber number based on ambient density
P = ambient pressure, atm.
V_j = jet velocity, cm./sec.
V_c = critical velocity, cm./sec.

Greek Letters

$\dot{\gamma}$ = shear rate, sec.$^{-1}$
μ = viscosity, poise

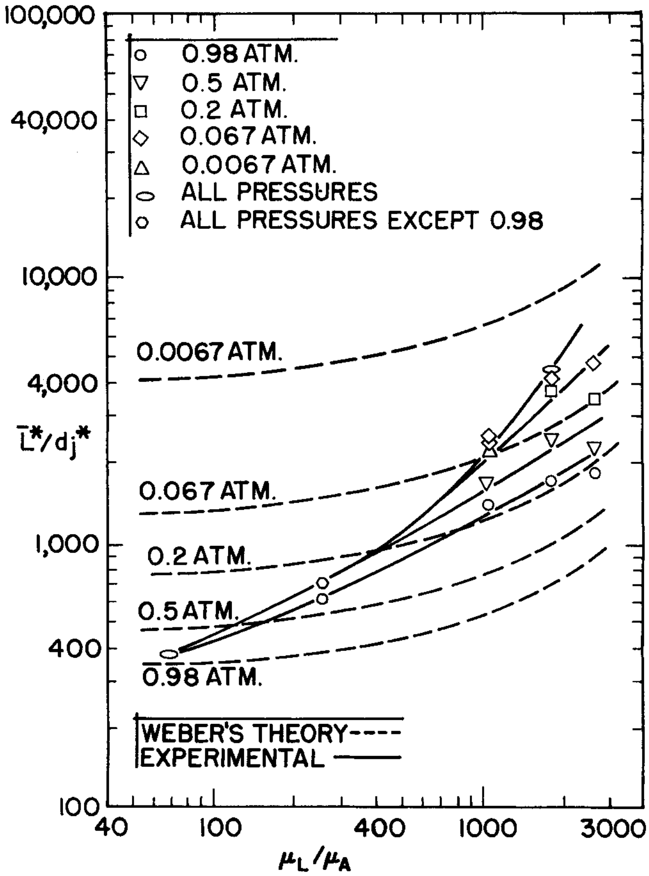

Fig. 11. Correlation for the maximum breakup length for all data.

ρ = density, g./cc.
σ = interfacial tension, dynes/cm.
χ = d_j/d_t = jet contraction ratio

Subscripts and Superscripts

A = ambient medium
L = liquid (jet)
$*$ = the maximum in the breakup curve

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Viscoelastic Jet Stability

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The breakup of a low speed horizontal jet is investigated. Weber's theory for the Newtonian jet is extended to a linear viscoelastic fluid. The theory predicts a dependence of breakup length on the elasticity number. Breakup lengths are measured for low concentration solutions of polyisobutylene in tetralin. Two molecular weights, several concentrations, and five capillary diameters were studied. A single correlation is obtained for all data which gives the breakup length as a function of the elasticity number, and the parameters of Weber's theory. At constant values of the Ohnesorge number and Weber number, the breakup length decreases with increasing elasticity number. The effect of the length of the capillary is studied. At large elasticity numbers short tubes give rise to slightly shorter breakup lengths than long tubes under identical flow conditions.

This paper, one of a series (*3, 4, 10*) on the stability of liquid jets, is especially concerned with jets formed by extrusion of viscoelastic liquids from long capillaries into still air. An investigation of the role played by elasticity in altering the growth rate of infinitesimal disturbances has been conducted, and the experimental results can be rationalized through a simple viscoelastic stability analysis.

The phenomenon studied is the breakup into droplets of a horizontal laminar cylindrical jet. The primary measurement is of the breakup length, L, the distance from the capillary exit to the point where the jet is no longer coherent. Earlier studies (*4*) using Newtonian fluids have shown that data may be correlated by Weber's theory (*13*), in the form

$$L/D = C_1 \, N_{We}^{1/2} \, (1 + 3Z) \tag{1}$$

The Ohnesorge number, Z, is the ratio of $N_{We}^{1/2}$ to N_{Re}, and is a measure of the relative importance of viscosity

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